

# 1 Mathematical Logic

**1.** On a very special island, there are two types of people: knights and knaves. Knights always tell the truth, while knaves always lie. On the island, we encounter two inhabitants, Tom and Peter.

Peter says: "Tom is a knave."

Tom says: "We are both knights."

What can we deduce about Peter?

**2.** On a very special island, there are two types of people: knights and knaves. Knights always tell the truth, while knaves always lie. We listen to a conversation between two islanders, Zig and Zan.

Zig: "It is not true that Zan is a knave."

Zan: "Zig and I are not the same type."

Who is a knight and who is a knave?

**3. (exam, November 2021)** On a very special island, there are two types of people: knights and knaves. Knights always tell the truth, while knaves always lie. On the island, we meet two inhabitants, Blaž and Luka.

Blaž says: "Luka is a knight."

Luka says: "We are both knights."

What is Blaž?

**4.** Exactly one of the chests labeled  $A$ ,  $B$ ,  $C$ , and  $D$  contains gold, and exactly one of the following statements is true. In which chest is the gold?

$A$ : The gold is here.

$B$ : The gold is in  $A$  or  $D$ .

$C$ : The gold is not here.

$D$ : The gold is here.

**5.** Knights always tell the truth, while knaves always lie. Andrej, Bine, and Cene are either knights or knaves. Determine who is a knight and who is a knave based on the following statements:

Andrej: "Bine is a knave, and Cene is a knight."

Bine: "If Andrej is a knave, then Cene is also a knave."

Cene: "I am not a knave, but at least one of the others is a knave."

**6.** On a very special island, there are two types of people: knights and knaves. Knights always tell the truth, while knaves always lie.

Today is dance day on the island, which means people are standing in circles, holding hands, and dancing. It is a time of joy, and it is especially difficult to distinguish knights from knaves.

Four individuals, namely Nika, Mikhail, Leon, and Timotej (in cyclic order), are holding hands in a circle. It appears that each of them says:

"I am in contact with exactly one knight!"

Is this possible? If so, find at least one arrangement of knights and knaves among the four individuals. Can you find a solution where Mikhail is a knave? Find all solutions!

**7.** Assume that the following statements are true:

- Pinocchio always lies;
- Pinocchio says: "All my hats are green."

Can we deduce that any of the following statements is true?

*A:* Pinocchio has exactly one hat, which is green.

*B:* Pinocchio has at least one green hat.

*C:* Pinocchio has no green hats.

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.